ANNALS OF THE UNIVERSITY OF ORADEA
FASCICLE OF TEXTILES, LEATHERWORK

THE FOURIER SERIES USED IN ANALYSE OF THE CAM MECHANISMS FOR THE SHOEMAKING MACHINES (PART II)

IOVAN-DRAGOMIR Alina [1], DRIȘCU Mariana [2]

[1, 2] „Gheorghe Asachi" Technical University, Textile, Leather and Industrial Management Faculty, Str.Prof.Dr.Doc. Dimitrie Mangeron, nr.28, 70050, Iasi, Romania, E-Mail: adragomir@tex.tuiasi.ro

Corresponding author: Iovan-Dragomir, Alina, E-mail: adragomir@tex.tuiasi.ro

Abstract: *A computer assisted procedure for the cinematic analysis of the mechanism of a cam is essential in making a certain type of research operations. They mainly refer to the optimization of operations running on specific machinery, or to the re-design of the mechanism, in order to make the mechanism digital. This analysis seems even more important, when we consider the fact that most of the machines used in shoe industry nowadays use a cam mechanism.*
The paper is devided in two parts.
In first part, it is elaborated a method of finding of a function G(x), belonging to a Fourier series, which approximates the numerical values $\{x_i, y_i\}$*, with the biggest accuracy. Finding the function that approximates the most accurately the data set, for the position parameters of the follower* $S(\omega)$*,* $\Psi(\varphi)$ *will lead to a complete kinematic and dynamic analysis of the cam mechanism. These values repeat with* $T = 2\pi$ *period.*
In second part, the method is tasted using MatCAD work sessions which allow a numerical and graphical analysis of the mathematical relations involved, in order to test the reability of the method. The set of experimental data are resulted after measuring a cam mechanism of a machine used in shoemaking.

Key words: *Fourier series, cam, machine, shoemaking*

1. INTRODUCTION

The kinematic analysis [1] of a cam mechanism aims to determine the position and kinematic parameters, the low of the movement, mechanism design features and operating phases.

First, it is determined the numerical correlation between the rotation angle of the cam and the position parameter of the follower. Then, the low of motion of the follower is derided, in order to calculate the velocity and acceleration of the follower.

This paper uses a set of experimental data that resulted after measuring a cam mechanism of a machine used in shoemaking industry, in order to prove the reliability of the methodology develop in part I using Fourier series.

2. GENERAL INFORMATION

2.1 Experimental appreciation of the methodology

For the experimental appreciation of the elaborated methodology it was used the software MathCAD system which allows a numerical and graphical analysis of the mathematical relations involved.

For example, one considers a series of experimental data (Φ_i, S_i), which are processing with the following routines [2], [3], [4]:

1. One creates the vectors of the initial data. In this example, the initial data are the angles Φ_i for the range $[0,2\pi]$ and length of a cam notated as S_i, mm.

2. For the appreciation of the elaborated methodology and the correctness of the data, the graph of the initial values is plotted in orthogonal coordinates, as can be seen in the Fig.1 and pollar coordinates, Fig.2, [3], [4], [5].

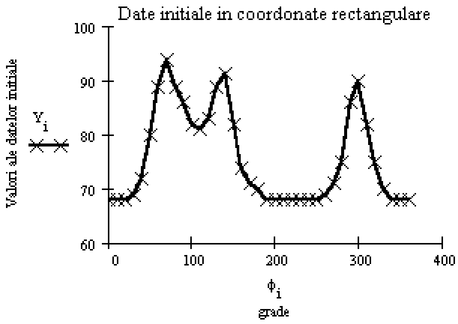

Fig. 1*: The graph of the initial value in ortogonal coordinates*

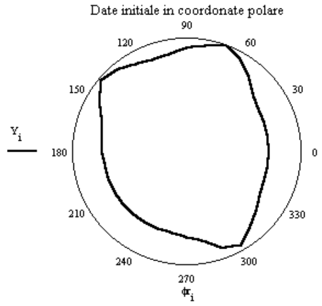

Fig. 2: *The graph of the initial value in pollar coordinates*

3. Using **MathCAD**, one can calculate with the relations the coefficients of the Fourier series a_0, a_i, b_i for those 37 values (Φ_i, S_i), and Fourier functions [6], [7], [8]:

$$F_i := \frac{a_0}{2} + \left[\sum_{n=1}^{nf} \left(a_n \cdot \cos(n \cdot \Phi r_i) + b_n \cdot \sin(n \cdot \Phi r_i) \right) \right] \quad \textbf{(1)}$$

where:

- i is index of those 37 experimental data;
- Φ_{ri} - angle for the range $[0, 2\pi]$, rad;
- N – degree of the Fourier coefficients;
- a_0, a_n and b_n - the Fourier coefficients;
- nf – the terms number of the Fourier series corresponding to the function $\mathbf{F_i}$.

2.2 Choosing of the number of coefficients a_n and b_n

The determination of that function approximating the initial data set with the best accuracy one proceeds as following:

1. One creates a matrix $\mathbf{G_{k,i}}$ of Fourier functions of 37 columns and *nf* lines like this:

$$G_{k,i} := \frac{a_0}{2} + \left[\sum_{n=1}^{k} \left(a_n \cos(n \cdot \Phi r_i) + b_n \cdot \sin(n \cdot \Phi r_i) \right) \right] \quad \textbf{(2)}$$

2. An analytical and graphical analysis will be made for the obtained functions. In this paper, for exemplification, the graph of the Fourier functions is presented for all points Φ_i=0°, 10°, 20° , 30°..360° and k=0, 5, 10..35, 36 (fig.3). Analyzing the graphical form, it results that the numerical data are approximated with a good accuracy with the Fourier functions defined for k=5, 10..35.

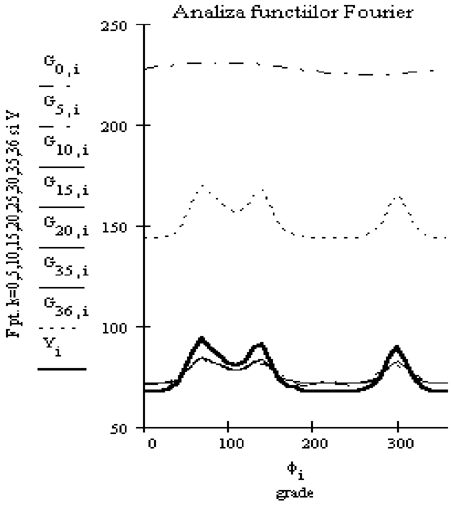

Fig.3: The graph of the Fourier function for all points for all points

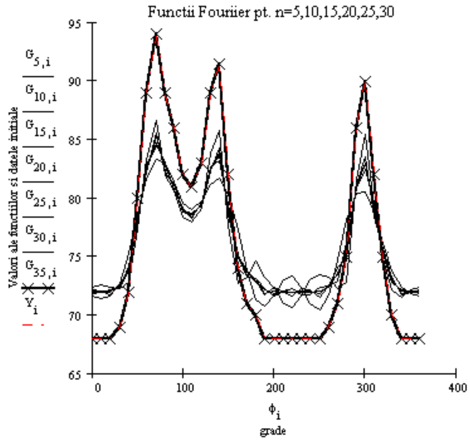

Fig.4: The graph of the Fourier function pentru n=10, 15, 20, 25,30 termeni

3. One limits the field of the Fourier functions for k=5, 10..35 and the graphical form of the Fourier functions is re-analyzed by comparison with initial graph (Fig.4). For numerical determination of that function which approximates with best accuracy initial data, a matrix of the errors will be created having the form: $\mathbf{ER_{k,i}=| G_{k,i}-Y_i|}$
in which will be inscribed the differences between initial data and those obtained by approximation, i=0...36, for those 36 Fourier functions, then one creates the vector V: $\mathbf{V_i=min\{ER_{k,i}\}}$

V =

	0
12	$2.132\cdot10^{-13}$
13	$8.527\cdot10^{-14}$
14	$2.274\cdot10^{-13}$
15	$9.948\cdot10^{-14}$
16	$2.842\cdot10^{-13}$
17	$7.105\cdot10^{-14}$
18	$1.421\cdot10^{-14}$
19	$1.847\cdot10^{-13}$
20	$3.837\cdot10^{-13}$
21	$1.137\cdot10^{-13}$
22	$1.99\cdot10^{-13}$
23	$1.563\cdot10^{-13}$
24	$5.4\cdot10^{-13}$
25	$3.553\cdot10^{-13}$
26	$1.563\cdot10^{-13}$
27	$2.132\cdot10^{-13}$

Fig.5: The analiz of the errors

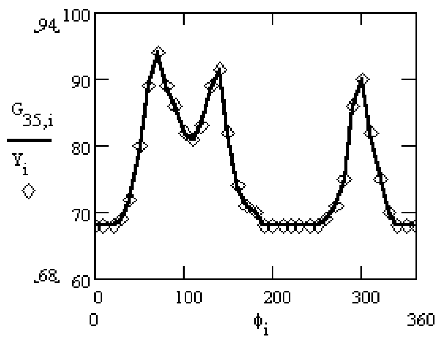

Fig. 6: The graph for the best Fourier function

For the data set studied one inscribes the values of the Fourier function defined of 35 terms in the vector V. Analyzing the numerical values inscribed in the vector V (Fig.5), one observes that they are very small, about 10^{-13} or 10^{-14}.

One can certainly assert that the Fourier function defined for 35 terms approximates with the best accuracy the initial data set.

The determined Fourier function will be:

$$F(x) := \frac{a_0}{2} + \left[\sum_{n=1}^{35} \left(a_n \cdot \cos(nx) + b_n \cdot \sin(nx) \right) \right] \quad \textbf{(3)}$$

defined for 36 values of the coefficients a_n and b_n for n=0,1,2..35

The graph of the Fourier function calculated for 36 terms *a*, *b* and initial point Yi is presented in the fig. 6. The analysis of this graph shows the mode of the positioning of the initial values *Yi* on the Fourier graph. The values were obtained in the session of getting data.

3.CONCLUSIONS

From those above presented one can assert doubtlessness that the Fourier function belonging to a Fourier series, having 35 terms approximates with the best accuracy a periodical experimental data set.Once one finds the function of the position parameters of the S peg and the other sets, that best approximates the set of data, a complete cinematic analysis of the cam can be made.

This analysis seems even more important, when we consider the fact that most of the machines used in textile industry nowadays use a cam mechanism. Taking into consideration the fact that the peg always executes an alternative and periodical movement, the procedure one has to use Fourier series to mathematically model the position parameters following the shift of the S peg in second part.

REFERENCES

[1] D.B.Marghitu - editor, *Mechanical Engineer's Handbook*, ACADEMIC PRESS, USA, 2001.

[2] D.F. Croitoru, A. Dragomir, *Utilaje pentru industria confecţiilor din piele,* Iasi, 2003.

[3] C. Oprișan, G.A. Popovici, *Mecanisme*, Editura TEHNICA-INFO, Chișinău, 2001

[4] E. Pintilie, N.Hoblea, *Elemente de proiectare a mașinilor pentru confecții textile*, Editura PERFORMANTICA, Iași, 2005

[5] A. Iovan-Dragomir, *Utilaje pentru încălțăminte*, Editura PERFORMANTICA, 2013.

[6] H. Dym, H.P. Mckean, *Fourier Series and Integrals,* Academic Pr., 1986.

[7] J.Ward Brown, V.Rull, *Fourier Series,* Churcill, McGraw_hill, 2001.

[8] B. Osilenker, *Fourier series in orthogonal polynomials*, Mathematical Reviews, 2001.